

The Contribution of a Wide Range of Space and Time Scales  
to the Northward Flux of Westerly Momentum

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The focus of this talk will be on the contribution of a wide range of time scales to the long-time average of atmospheric variances and covariances, and in particular on the contribution of the annual cycle. In addition, the interannual variability of seasonally averaged fluxes will be analyzed in the same context. For definiteness, numerical results will be shown for the eddy momentum flux at 200 mb as obtained from seven years of NMC analyses (March 8, 1970 - March 10, 1977).

Any discussion of variability encompassing time scales of longer than one month should address the question of the role of the seasonal cycle. One way of defining the seasonal cycle is as the average value of any atmospheric variable as a function of the time of year. The seasonal cycle represents the completely predictable component of the general circulation, predictable on the basis of past data alone. What is its contribution to the long-term averaged eddy momentum flux?

The very existence of the seasonal cycle renders atmospheric time series non-stationary, and this causes the traditional separation between time mean and departures therefrom to be quite misleading when applied to records whose length is one season or greater. For instance, consider a hypothetical time series whose seasonal cycle resembles a parabola over the course of one season (see Figure 1). If we define the transients as departures from this smooth seasonal cycle, they represent fluctuations about the completely predictable component. Comparison of this definition of transients with the traditional one of departures from the time mean (Figure 1) shows that the traditional definition includes contributions due to the seasonal cycle, contributions which ought to be separated from the less predictable fluctuations.

In mathematical terms, let  $A(t)$  and  $B(t)$  be any two atmospheric time series,  $\hat{A}(t)$  and  $\hat{B}(t)$  their seasonal cycle components, and  $A'(t)$  and  $B'(t)$  the fluctuations about the seasonal cycle components:  $A'(t) = A(t) - \hat{A}(t)$ ,  $B'(t) = B(t) - \hat{B}(t)$ . Letting an overbar denote a time average over the length of the record, the time averaged product  $\overline{AB}$  is  $\overline{AB} = \overline{\hat{A}\hat{B}} + \overline{A'B'} + \overline{\hat{A}B'} + \overline{A'\hat{B}}$ . In order to unambiguously ascribe a certain fraction of the total covariance to the seasonal cycle and a certain fraction to the transients, it is necessary to guarantee the orthogonality of these two components over the record length, so that the cross terms vanish. (This orthogonality holds automatically if one uses the traditional definition of transients, namely  $A'(t) = A(t) - \bar{A}$ , for then  $\overline{AB} = \overline{A'B'} + \overline{AB}$ .)

When the record length  $T$  is an integer number of years (here  $T = 7$  years), there is a natural solution to the orthogonality problem. This involves the use of Fourier analysis:

$$A(t) = \bar{A} + \sum_m (A_m \cos(2\pi t_m/T) + B_m \sin(2\pi t_m/T)).$$

The  $m$ th term in the series describes fluctuations with a period of  $T/m$ . Based on this decomposition, it is possible to divide  $A(t)$  in four pieces:

$$A(t) = A_{SC}(t) + A_{ULF}(t) + A_{LF}(t) + A_{MF}(t)$$

$A_{SC}(t)$  is the seasonal cycle component, and consists of the time mean  $\bar{A}$  plus the annual ( $m=7$ ) and semiannual ( $m=14$ ) Fourier components.  $A_{ULF}(t)$  is the ultra-low frequency component, and consists of fluctuations with periods longer than one year ( $m < 7$ ).  $A_{LF}(t)$  is the low frequency component, and consists of fluctuations with periods less than one year but greater than 7.20 days, excluding the semi-annual period ( $8 < m < 355$ ,  $m \neq 14$ ).  $A_{MF}(t)$  is the medium frequency component, and consists of fluctuations with periods between 7.20 days and 2.0 days, the Nyquist period ( $356 < m < 2560$ ). Because of the orthogonality of the Fourier components over the seven year record length,

$$\begin{aligned} \overline{A(t) B(t)} &= \overline{A_{SC}(t) B_{SC}(t)} + \overline{A_{ULF}(t) B_{ULF}(t)} \\ &+ \overline{A_{LF}(t) B_{LF}(t)} + \overline{A_{MF}(t) B_{MF}(t)}. \end{aligned}$$

The seven year averaged eddy momentum flux is thus a sum of separate fluxes, one for each term, with no cross terms. A comparison of the magnitude of the four terms (Figure 2) shows that the seasonal cycle and low frequency components together contribute roughly 70%-80% of the total. The seasonal cycle component alone is significant.

These ideas are now applied to the interannual variability of the fluxes. If we want to retain the high degree of time scale separation afforded by the Fourier basis set, and yet be able to examine seasonal (rather than long-term) averages, we lose the orthogonality property and must deal with non-zero cross terms. Combining the ultra-low and low frequency categories into a new low frequency category, and denoting a time average over a single season with a double overbar, we have:

$$\begin{aligned} \overline{\overline{A(t) B(t)}} &= \overline{\overline{A_{SC} B_{SC}}} + \overline{\overline{A_{LF} B_{LF}}} + \overline{\overline{A_{MF} B_{MF}}} + \\ &\quad (\overline{\overline{A_{SC} B_{LF}}} + \overline{\overline{A_{LF} B_{SC}}}) + (\overline{\overline{A_{SC} B_{MF}}} + \overline{\overline{B_{SC} A_{MF}}}) + (\overline{\overline{A_{LF} B_{MF}}} + \overline{\overline{A_{MF} B_{LF}}}). \end{aligned}$$

The cross terms arise because the different components are not orthogonal over a given season.

The seasonal eddy momentum flux and its contributing terms were examined for the seven winter seasons, and the results shown in Figures 3-10. The seasonal cycle contribution is not shown on these graphs, since it is the same every winter and hence cannot contribute to interannual variability. The interaction terms between the medium frequency component and the other components were very small, and are not shown. The interaction between the seasonal cycle and low frequency components is, however, quite important. On the average it seems to explain more of the year to year variation than any of the other components.

This suggests that the interannual variability of the winter eddy momentum flux at 200 mb is due to distortions of the climatological flow which, although they may not transport a significant amount of momentum themselves, interact with the normal (or climatological) component to transport anomalous momentum.

### Figure Captions

- Figure 1. Hypothetical time series (solid line), the seasonal cycle based on that time series (dashed line), and the seasonal mean (dotted line).
- Figure 2. Contributions to long time averaged 200 mb eddy momentum flux as a function of latitude. The seasonal cycle contribution is the heavy solid line, the low frequency contribution is the heavy dashed line, the medium frequency contribution is the light solid line, and the ultra-low frequency contribution is the light dashed line.
- Figure 3. Contributions to the winter averaged 200mb eddy momentum flux at 62°N as a function of year. The total winter flux is shown by the heavy solid line, the contribution due to interactions between low frequency and seasonal cycle components by the heavy dashed line, the contribution due to low frequency components alone by the light solid line, and the contribution due to medium frequency components alone by the light dashed line.
- Figure 4. Contributions to the winter averaged 200 mb eddy momentum flux at 56°N as a function of year. As in Figure 2.
- Figure 5. Contributions to the winter averaged 200 mb eddy momentum flux at 50°N as a function of year. As in Figure 2.
- Figure 6. Contributions to the winter averaged 200 mb eddy momentum flux at 44°N as a function of year. As in Figure 2.
- Figure 7. Contributions to the winter averaged 200 mb eddy momentum flux at 38°N as a function of year. As in Figure 2.
- Figure 8. Contributions to the winter averaged 200 mb eddy momentum flux at 32°N as a function of year. As in Figure 2.
- Figure 9. Contributions to the winter averaged 200 mb eddy momentum flux at 26°N as a function of year. As in Figure 2.
- Figure 10. Contributions to the winter averaged 200 mb eddy momentum flux at 20°N as a function of year. As in Figure 2. The contribution due to interactions between low frequency and seasonal cycle components has had  $10 \text{ (m/sec)}^2$  added to it.

— Time Series.  
 --- Seasonal Cycle  
 ..... Time Mean

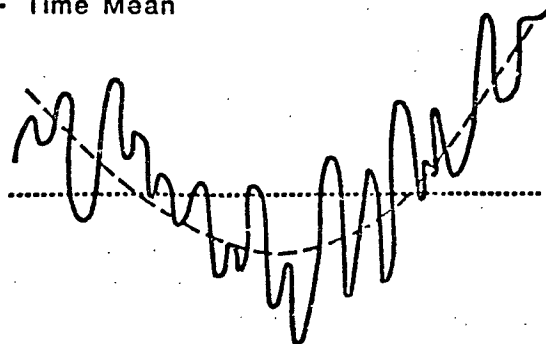


Figure 1

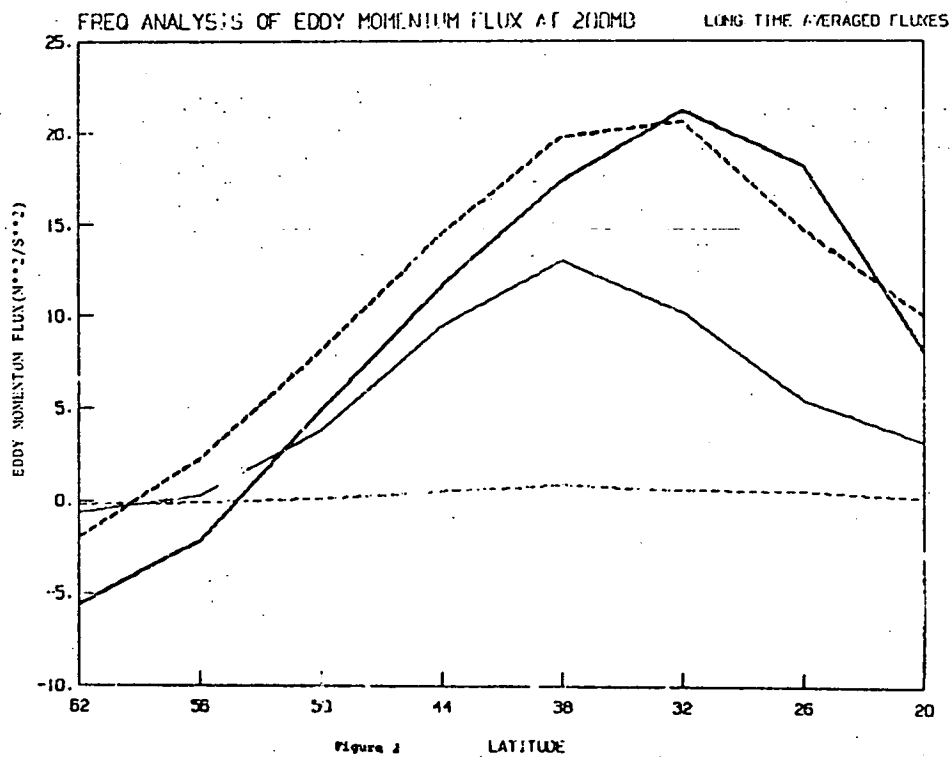


Figure 2

